INFLUENCES OF SPACE CHARGE EFFECT DURING ION ACCUMULATION USING MOVING BARRIER BUCKET COOPERATED WITH BEAM COOLING*

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Abstract

A longitudinal ion storage method by using a moving barrier bucket with a beam cooling can accumulate the ions in a storage ring, effectively. After the multicycle injections of the beam bunch by the method, the space charge effect due to the stored particles can interfere the next accumulation of the ions, because the space charge potential can cancel the effective barrier voltage. Using numerical simulations, we employ the longitudinal particle tracking, which takes into account the barrier bucket voltage, the beam cooling and the space charge effect, for the study of the beam dynamics during the accumulation operations. As a result, it is found that the space charge effect limits the accumulation of the ions in the longitudinal storage method.

INTRODUCTION

Longitudinal beam stacking by using a moving barrier bucket system with a stochastic momentum cooling has been proposed [1]. In the proposal, not only the stochastic cooling was applied, but also the electron cooling can be a candidate for the operation [2]. The ion storage experiment by using a barrier bucket with the electron cooling has been carried out, and the experimental results are succeeded for the ion beam stacking [3].

During the ion beam stacking, the beam current will be increased with the injection numbers. The high current beam can create the strong space charge potential. The electric field induced by the space charge may interferer the large number of the bunch injections and the higher stacking ratio.

In this study, we developed the longitudinal particle tracking code with the space charge effect, and the beam dynamics is numerically investigated by using the developed code. The numerical simulation results indicate the limitation of the ion accumulation derived from the self electric field in the stacking method. Also the space charge effect can be predicted by the simple ellipsoid shape model, and it is useful to estimate the stacking limit.

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OPERATION OF ION ACCUMULATION BY MOVING BARRIER BUCKET WITH ELECTRON COOLING

The longitudinal ion accumulation by using the moving barrier bucket with the electron cooling can be operated as follows [2, 4]. First, the bunch is injected into the region between two barrier voltages. The energy spread of the beam is decreased by the electron cooling. After the cooling, the beam with the small energy spread is separated by the moving barrier bucket operation for the partitioning. To repeat the above procedure, the ions are accumulated with the new injections.

LONGITUDINAL PARTICLE TRACKING OPERATED BY MOVING BARRIER BUCKET WITH SPACE CHARGE EFFECT

Basic Equations of Motion in Phase Space

The energy difference $\Delta E = E - E_s$ [eV/n] from the synchronous energy E_s in the barrier bucket is calculated by

$$\frac{d\Delta E}{dt} = \frac{q}{m}\frac{V_{bb}}{T_0} - E_{cool} - \frac{q}{m}\frac{g}{4\pi\epsilon_0\gamma^2}\frac{d\lambda}{d\tau}. \quad (1)$$

where q is the charge state of the beam ion, m is the atomic mass number, $V_{bb} \equiv V_{bb}(t,\tau)$ is the voltage of the moving barrier bucket, T_0 is the revolution period, E_{cool} is the beam cooling term, g is the geometry factor, ϵ_0 is the permittivity of free space, and λ is the line charge density.

The time τ in the moving frame depends on time t in the laboratory frame is calculated by

$$\frac{d\tau}{dt} = \frac{\eta}{\beta^2}\frac{\Delta E}{E_0},$$

where β is the velocity divided by light speed c, $\eta = 1/\gamma^2 - 1/\gamma_{tr}^2$ is the phase slip factor with the transition gamma γ_{tr}. Here $E_0 = E_k + m_0c^2$ is the synchronous energy per nucleon, where E_k is the kinetic energy per nucleon and $m_0c^2 = 931.481$ MeV is the rest energy of the atomic mass unit based on ^{12}C.

Barrier Bucket Voltage

Figure 1 shows the barrier bucket voltage waveform at each injection time. The barrier bucket shape is a sinusoidal waveform, and the pulse duration T_1 is 200 ns. The duration between the left and right barrier pulses $T_2 \equiv T_2(t)$

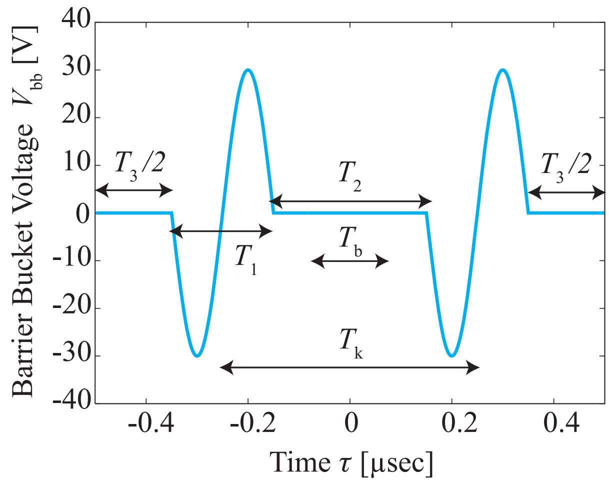

Figure 1: Barrier bucket voltage waveform at each injection time.

is changed as $-T_1/2 \sim 300$ ns. The duration of the storage region $T_3 \equiv T_3(t)$ is > 300 ns. The flat-top region of the magnetic kicker pulse T_k is 500 ns. The injection of new batch is carried out in the duration $T_b = 150$ ns. The maximum amplitude of the barrier bucket voltage is 30 V in this paper. According to the above voltage waveform, the kinetic energies of the ions are changed as the first term of the right hand side in Eq. (1).

Electron Cooling

The electron cooling term (second term of the right hand side in Eq. (1)) is solved by

$$E_{cool} = -\Delta E\, k_c G,$$

for the longitudinal direction, and $d\,\varepsilon_t/dt = -2k_c\,\varepsilon_t\,G$, for the transverse emittance ε_t. Here

$$G = \left[\frac{\beta^2\gamma^2\varepsilon_t}{\beta_c} + \left(\frac{\Delta E}{\beta E_s}\right)^2 + \frac{2\,T_{eff}}{m_e c^2}\right]^{-3/2},$$

and the coefficient is

$$k_c = \frac{4r_e r_n c\, n_e \eta_c L_p}{\gamma^2}\frac{q^2}{m},$$

where β_c is the beta function at the cooler section, T_{eff} is the effective temperature of the electron beam, m_e is the electron mass, r_e and r_n are the classical electron and proton radii, n_e is the number density of the electron beam, and L_p is the Coulomb logarithm. Here $\eta_c = L_{ec}/C$, where L_{ec} is the cooler length and C is the circumference of the ring.

Space Charge Effect

The calculation for the space charge effect is based on a particle-in-cell (PIC) method [5]. The particles give the charge to the grid points, and the line charge density can be calculated at each grid. According to the third term of the right hand side in Eq. (1), the space charge effect can be included in the longitudinal particle dynamics.

ESTIMATION OF SPACE CHARGE POTENTIAL BY SIMPLE BUNCH MODEL

When a beam bunch has the uniform density with the ellipsoid shape, the space charge potential can be calculated by an analytical formula. The maximum potential of the ellipsoidal bunch in free space is given as [6]

$$|\phi_{fs}| = \frac{\rho_0}{2\epsilon_0} M_E {z_b}^2, \tag{2}$$

where ρ_0 is the uniform charge density and the factor M_E is derived by

$$M_E = \frac{1-\xi^2}{\xi^2}\left(\frac{1}{2\xi}\log\frac{1+\xi}{1-\xi} - 1\right).$$

Here $\xi = \sqrt{1 - r_b^2/z_b^2}$, where r_b is the beam radius and z_b is the bunch half length in the longitudinal direction. The charge density is calculated by

$$\rho_0 = \frac{qeN_b(N_{inj}-1)}{V_b},$$

where e is the elementary charge, N_b is the number of ions per batch, N_{inj} is the injection number, and the volume of the ellipsoid is written by $V_b = 4\pi {r_b}^2 z_b/3$. The bunch half length is estimated by $z_b = T_3 C/2T_0$.

From Eq. (2), the potential is calculated by $|\phi_{fs}| = 31.4$ V ($> V_{bb}$) after 16 injections, and the space charge potential overcomes the barrier bucket voltage. The parameters for the estimate are as follows:

Table 1: Bunch parameters

Ring circumference C	108.36 m
Revolution time T_0	1000 ns
Beam radius r_b	0.1 mm
Charge state q	18
Number of Ions per batch N_b	7×10^7

NUMERICAL SIMULATION RESULTS

We numerically simulate the beam dynamics in the moving barrier bucket with a momentum cooling process by using the procedures described in the previous section. The example calculation is performed by using data of the last ESR experiment [3]. The condition is summarized as Table 2.

The injected ions per batch are represented by 500 particles in a manner as PIC method, and the ions have a Gaussian distribution as the energy spread and a uniform distribution in the time at the injection time. The longitudinal

Table 2: Parameters for numerical simulations

Beam	
Ion species	$^{40}Ar^{18+}$
Kinetic energy	65.3 MeV
Particle number per batch	7×10^{7}
Energy spread (1σ) of injection batch	0.3 MeV
Ring	
Circumference C	108.36 m
Energy acceptance	+/- 0.6 MeV
Phase slip factor η	-0.6959
Electron cooler	
Electron beam current	0.3 A
Effective temperature T_{eff}	10^{-3} eV
Cooler length L_{ec}	1.8 m
Beam diameter	50 mm
Beta function β_c	16 m

phase space is discretized by 50 grids for the space charge calculation. A new injection bunch is added in the particle tracking calculation at each injection time, and the particles in the region of the kicker magnet operation become the lost particles before the injection procedure. The geometry factor is assumed to be $g = 2$, and the Coulomb logarithm is $L_p = 2$. The initial transverse emittance at each injected ion ε_t is set by 1.36 mm mrad.

Figure 2 shows the typical particle distributions, the barrier bucket voltage waveforms and the space charge potentials. The stacked ions are separated by the moving barrier bucket voltage, and the ions are accumulated into T_3 region. The stacked ions generate the space charge potential.

The longitudinal emittance during the operation is shown in Fig. 3. The longitudinal emittance ε_z is given by

$$\varepsilon_z = \left[\left\langle(\Delta E - \Delta E_0)^2\right\rangle \left\langle(\tau - \tau_0)^2\right\rangle - \langle(\Delta E - \Delta E_0)(\tau - \tau_0)\rangle^2\right]^{1/2},$$

where ΔE_0 and τ_0 are the average values. After the beam injection into the storage ring, the longitudinal emittance decreases rapidly, because the energy spread can be decreased by the electron cooling. The cooled beam is separated by the barrier bucket to the accumulation region T_3, and the longitudinal emittance is slightly increased by the operation. The emittance increases due to the new beam bunch injected with the large energy spread. Since the stacked ions can be continued cooling by the electron cooler, the fluctuations of the longitudinal emittance become comfortable into some level.

The accumulated ion number normalized by the number of ions per batch is shown in Fig. 4, and Figure 5 shows the ratio of the stacked ions in the ring to the number of injected ions. Although the number of the accumulated ions increases with the new bunch injections, the particle number is saturated as shown in Fig. 4. As shown in Fig. 5, the stacking efficiency is rapidly decreased after 150 sec.

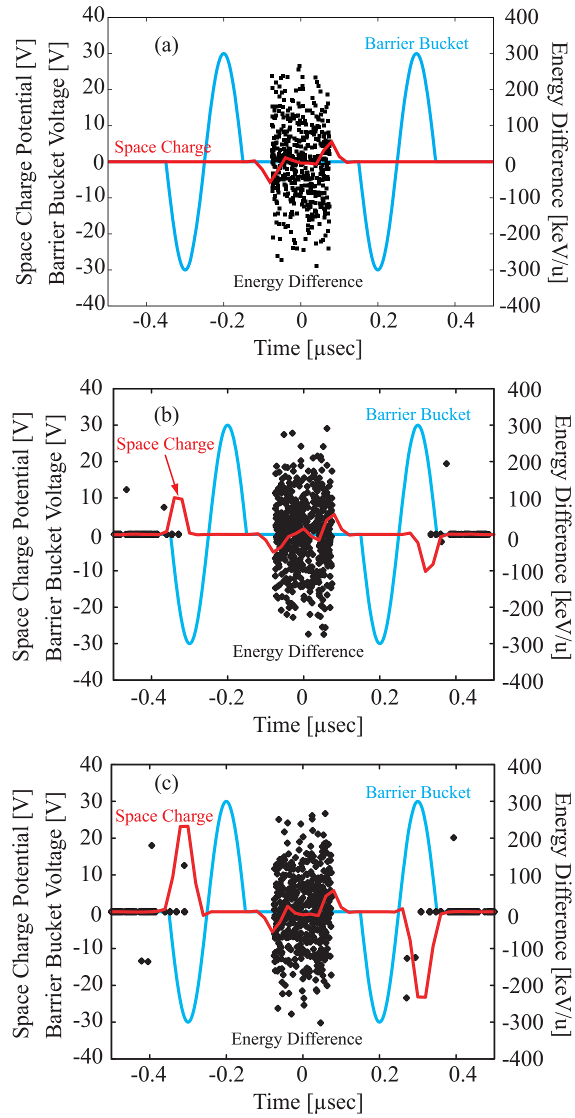

Figure 2: Particle distribution (black dot) in $\Delta E - \tau$ phase space and barrier bucket voltage (cyan line) and space charge potential (red line), (a) for initial condition (1st injection), (b) for 6th injection, and (c) for 21th injection.

The space charge potential generated by the accumulated ions at each injection number is shown in Fig. 6. Figure 7 shows the amplitude of the barrier bucket voltage and the maximum values of the space charge potential created by the stacked ions at each stacking number $N_{inj} - 1$. The maximum value of the space charge potential is indicated as the numerical simulation results in Fig. 6. The solid curve in Fig. 7 is estimated by the ellipsoid bunch shape model in Eq. (2).

The space charge potential can cancel the barrier voltage, and the effective voltage of the barrier bucket decreases. For this reason, the stacked ions penetrate the barrier bucket region as shown in Fig. 2, and the barrier bucket cannot effectively control the ions due to the space charge effect. The ions penetrated inside the barrier bucket are kicked out

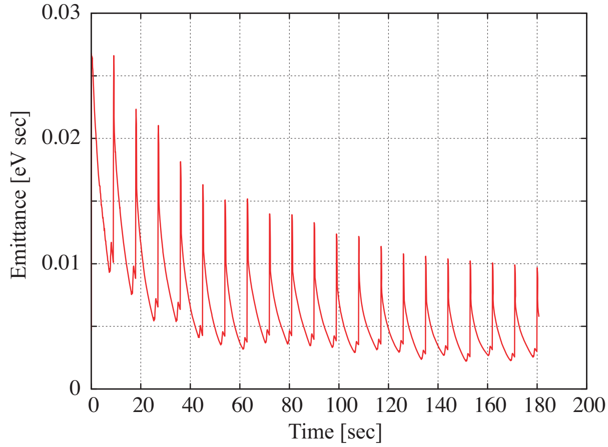

Figure 3: Longitudinal emittance history.

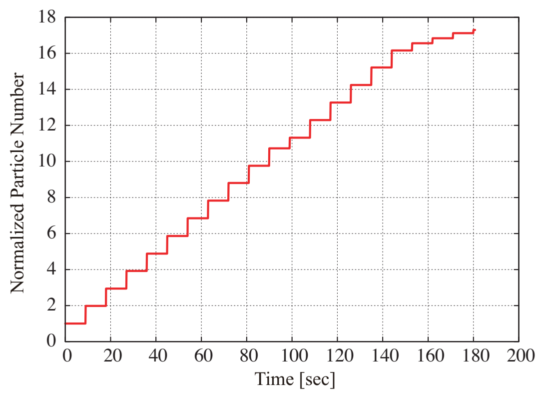

Figure 4: Stacked particle number normalized by the ions per injection batch.

from the ring at the next magnetic kicker operation.

CONCLUSIONS

Space charge effect during the ion accumulation using the moving barrier bucket cooperated with the electron cooling was numerically investigated by using the longi-

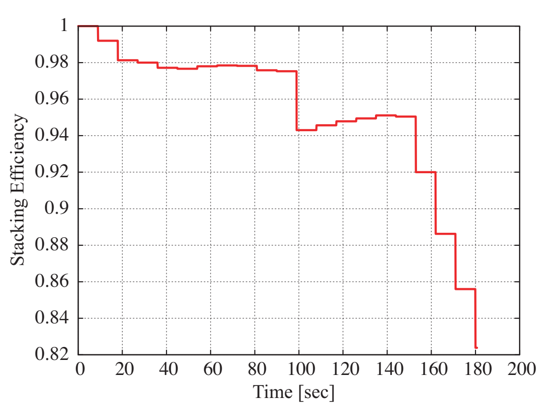

Figure 5: Stacking efficiency (= particle number in the ring / injected particle number).

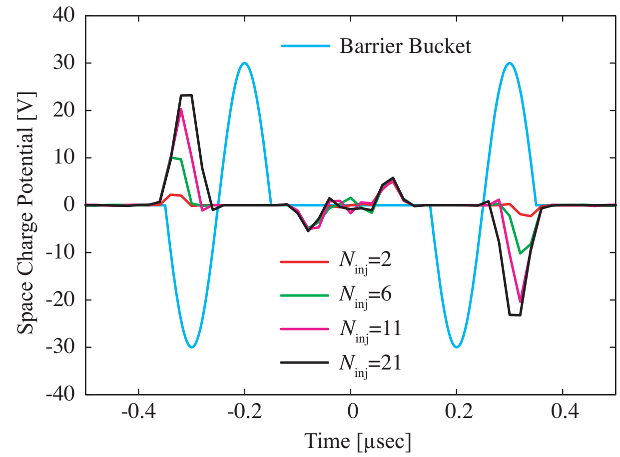

Figure 6: Barrier bucket voltage and space charge potential during the ion storage.

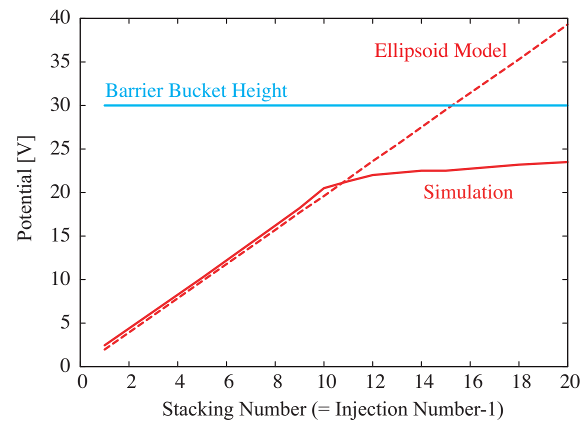

Figure 7: Space charge potential of the analytical estimation and the simulation result.

tudinal particle tracking. It was found that the ion accumulation can be interfered due to the space charge potential created by the stacked ions. Space charge effect is one of important roles for stacking antiprotons and ions in an accumulation ring. The space charge effect can be predicted by the simple ellipsoid shape model, and it may be useful to design the barrier voltage.

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