USE OF AN ELECTRON BEAM FOR STOCHASTIC COOLING*

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Abstract

Microwave instability of an electron beam can be used for a multiple increase in the collective response for the perturbation caused by a particle of a co-moving ion beam, i.e. for enhancement of friction force in electron cooling method. The low scale (hundreds GHz and higher frequency range) space charge or FEL type instabilities can be produced (depending on conditions) by introducing an alternating magnetic field along the electron beam path. Beams' optics and noise conditioning for obtaining a maximal cooling effect and related limitations will be discussed. The method promises to increase by a few orders of magnitude the cooling rate for heavy particle beams with a large emittance for a wide energy range with respect to either electron and conventional stochastic cooling.

INTRODUCTION

The high-energy cooling plays a critical role in raising the efficiency of existing and future projects of hadron and lepton-hadron colliders: RHIC with heavy ion and polarized proton-proton colliding beams [1] and electron-ion collider eRHIC [2,3] of Brookhaven National Laboratory; ELIC [2,4] of Jefferson Laboratory; the proton-antiproton collider of Fermilab; and, perhaps, even the LHC of CERN.

Electron cooling proved to be very efficient method of cooling intense hadron- and ion-beams at low and medium energies [5]. The electron cooler of 9 GeV antiprotons in the Fermilab recycler represents state-of-the-art technology [6] and already led to significant increase luminosity in the proton-antiproton collider. Development of the ERL-based electron cooler at BNL promises effective cooling of gold ions with energies of 100 GeV per nucleon [7].

Realization of effective cooling in hadron (proton, antiproton) colliders of higher energies requires new conceptual solutions and techniques. Currently, an ERL-based EC scheme is under study which includes a circulator-cooler ring as a way to reduce the necessary electron current delivered by an ERL [8]. It should be noted, that the ERL-based electron cooling should be operated in staged regime (cooling starts at an intermediate energy e.g. injection energy of a collider ring) to be continued in the collider mode after acceleration. Finally, for best performance of a collider an initial transverse stochastic cooling of a coasted beam should precede use of EC [8].

An extremely challenging character of high energy cooling projects for hadron beams quests to search for possible ways to enhance efficiency of existing cooling methods or invent new techniques.

It was noted by earlier works [9], that potential of an electron beam-based cooling techniques may not be exhausted by the classical electron cooling scheme. Namely, the idea of *coherent electron cooling* (CEC) encompasses various possibilities of using collective instabilities in the electron beam to enhance the effectiveness of interaction between hadrons and electrons. CEC combines the advantages of two existing methods, electron cooling (microscopic scale of interaction between ion beam and cooling media, the electron beam) and stochastic cooling (amplification of media response to ions). It is based on use of a co-transported electron beam in three roles – a receiver, amplifier and kicker. Such principle seems flexible for implementation in hadron facilities of various applications in a wide energy range from non-relativistic beams to beams in colliders.

Below we will review the CEC principles and limitations referring to earlier works [9] as well as recent work [10] which is specifically devoted to development of CEC system for colliding beams by use of SASE FEL as amplifier.

PREREQUISITES OF CEC

A General CEC Idea

The electron cooling-a method of damping the angular and energy spread of the beams of heavy charged particles- is, as known, [11-13] that the beam in the straight section of an orbit is passing through an accompanying electron beam having lower temperature. In this case, heavy particles are decelerated with respect to electron medium similarly to that as is occurred in usual plasma at Ti >Te .

A principle suggested here of an amplification is naturally inserted into logical scheme of the method. On the cooling section such conditions should be arranged that the moving "electron plasma" should become unstable in the given range of the wave lengths. Then, an excitation caused by an input ion will be transferred by electron flux developing exponentially independent of the ion; at the output from electron beam the ion acquires the momentum correlated with its input velocity (Figure 1).

A firm correlation between input and output signals is maintaned unless the excitation reaches the nonlinear regime, i.e. the density modulation within the required scale of distances remains relatively small. It is, of course, necessary to provide the optimum output phase relations in the position, and velocity of an ion with respect to electron "avalanche" produced by the ion. Such a task is facilitated by the motion of ions and electrons in the fields

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given are absolutely different. In particular, after interaction at the "input" the beams can be separated and then can be made interacting again at the "output".

Another important condition is the noise level in electron flux at the input i.e. the electron density fluctuations of which will also be increased is sufficiently small. We will discuss some possibilities to provide this condition.

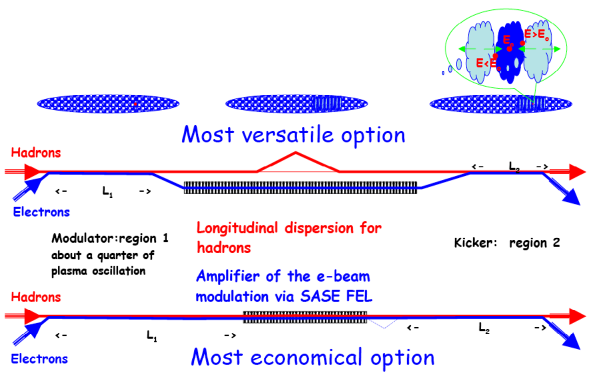

Figure1: Schematic layout of the CEC with three sections: a) A modulator, where the electron beam is polarized (density modulated) by presence of hadrons; b) A gain section, where density modulation in the electron beam is amplified; c) A kicker, where the amplified longitudinal electrostatic field in the electron beam accelerates or decelerates hadrons. [10]

Polarization of Plasma by a Fast Ion

Under condition that the spread of electron velocities is small compared to that of the ion beam, we will consider the electron beam as homogeneous, isotropic plasma of density n_e We then derive a hydrodynamic equation (in a co-moving frame) for perturbation of electron density, $\tilde{n}(\vec{r},t)$ by an ion of a charge Ze which arrives in the beam at initial moment $t=0$:

$$\ddot{\tilde{n}} + \omega_e^2 \tilde{n} = -Ze\omega_e^2 \delta(\vec{r} - \vec{\upsilon} t) \tag{1}$$

Here $\omega_e = \sqrt{4\pi n_e e^2 / m}$ is plasma oscillation frequency. Solution of this equation with zero initial conditions is as follows:

$$\tilde{n} = -Ze\omega_e \int_0^t \delta(\vec{r} - \vec{\upsilon}\tau) \sin \omega_e (t-\tau) d\tau \tag{2}$$

We then can find the related electric field:

$$\tilde{\tilde{\mathrm{E}}} = -Ze\omega_e \int_0^t d\tau \frac{\vec{r} - \vec{\upsilon}\tau}{\left| \vec{r} - \vec{\upsilon}\tau \right|^3} \sin \omega_e (t-\tau) \tag{3}$$

In particular, by taking this field along ion trajectory $\vec{r} = \vec{\upsilon} t$, we immediately obtain the well-known drag force of electron cooling, in this case associated with the collective response of electrons to a fast ion:

$$\vec{F} = -(Ze)^2 \omega_e^2 \frac{\vec{\upsilon}}{\upsilon^3} \int_{\tau_{\min}}^t \frac{d\tau}{\tau} \frac{\sin \omega_e \tau}{\omega_e \tau} \tag{4}$$

with $\tau_{\min}$ here should be equal to r_D/v.

Equations (2) through (4) clearly show the importance of taking into account plasma oscillations in dynamics of the collective electron response to the ion. It also should be noted that the solution (2) and (3) suggest a paradox: an ion does not perturb the electron density anywhere but only along its trajectory – yet creating a non-zero charge along its string wake! Apparently, the paradox is removed by taking into account the plasma boundaries where the excited electric field (3) creates the correspondent surface charge. In other words, one has to explicitly compute the polarization problem for an electron beam of finite transverse sizes. Other important factor will be electron beam non-isotropy connected to a focusing either solenoidal at low energies or quadrupole at high energies. Both these factors, non-isotropy and boundaries, will contribute in appearance of electron space charge perturbation distributed around the ion.

Possible Micro-instabilities of the Electron Beam

Electron polarization and the collective response to an ion could be increased proportionally to the number of electrons in the interaction region if the initial excitations could increase spontaneously. For this, the electron plasma should be able to self-bunching, i.e. should be unstable in the region of the wave-lengths exceeding the electron Debye distance $r_D = \Delta v_e / \omega_e$. There exist possibilities for a few different type of this scale microwave instabilities depending on energy and beam transport conditions [9,14].

A. A microwave Coulomb instability that seems easy to realize at low energies is the parametric instability of longitudinal plasmas oscillation of the electron beam; it occurs when plasma parameter of the electron beam ω_e is modulated with the frequency $\omega = 2\omega_e$. Such a modulation can be realized via modulation of electron beam size by varying strength of a solenoid in which the electron beam is immersed (magnetized).

B. The mechanism of instability with the properties required could be precluded if in the cooling section the transverse alternating magnetic field (undulator) of relatively small amplitude is introduced into the longitudinal magnetic field B_s accompanying an electron beam. For the sake of simplicity let us take this transverse field as a helically-variable; in the complex form:

$$B_x + iB_y = \alpha B_u \exp(iz / \lambda\!\!\!^{-}_u)$$

where x and y are transverse coordinates, z is the longitudinal coordinate, B_u and α are respectively the magnitude and angular deviation of a total magnetic field; $\lambda_u = 2\pi \lambda\!\!\!^{-}_u$ is a helical step. If the transverse size of an electron beam is small compared to $\lambda\!\!\!^{-}_u$, one can neglect the transverse inhomogeneity of the magnetic field. The field should be so large that the cyclotron frequency of electrons should significantly be larger than that of a plasma (the suppression condition of the space charge influence or magnetizing):

$$\Omega \equiv \frac{eB_s}{m_e c} >> \omega_e$$

where the frequencies are related to the beam rest frame. For a particle motion in such a field a constant parameter is an energy in laboratory frame or the total velocity $v=\beta c$, but the transverse and longitudinal velocity dependence of energy may change sign at sufficiently strong magnetic field. Then, instability of the *negative longitudinal mass* is realized in the region

$$\frac{1}{\mu}\equiv\gamma^3 m\frac{d\upsilon_z}{dp_z}=[1-\frac{(\gamma\beta\theta_u)^2}{\Delta}\frac{\lambda_c}{\lambda_u}]/(1+\frac{\theta_u^2}{\Delta})<0$$

$$\Delta\equiv 1-\frac{2\pi p_z c}{eB\lambda_0}\qquad \theta_u=\frac{\alpha}{\Delta}$$

i.e. when an average velocity becomes a decreasing function of energy because of an increase in the forced transverse velocity. When the resonance $\Delta=0$ is far enough ($\Delta>>\Lambda/\Omega$), one can neglect the transverse mobility of electrons; at this approximation the increment length (in laboratory frame) is given by formula

$$l_\mu=\frac{\gamma}{\sqrt{1+\gamma^2\theta_u^2}}\left|\frac{k\sqrt{\mu}}{k_z\omega_e}\right|$$

where $\vec{k}=(k_z,\vec{k}_\perp)$ is a wave vector of Fourier space harmonics of an excited electric field [9,14].

C. In ultra-relativistic region the mechanism of radiation instability can be effective [9] which is connected to generation of coherent radiation with wave lengths satisfying the following condition:

$$k-k_z v_z\approx k_0 v_z$$

The systems based on this principle acquired the name "free electron lasers" (FEL). The increment length of this instability is equal to (here we use a notation J for electron peak current and notation J_A for *Alfven current* $m_e c^3/e\approx 17kA$) [15,16]:

$$l_g\approx\lambda\!\!\!^{-}_u[\frac{\gamma J_A}{J(1+2\ln\frac{\sigma_{cr}}{\sigma_\perp})}]^{1/2}\qquad(5)$$

at $\sigma_\perp<<\sqrt{\lambda\!\!\!^{-} l_g}$, while

$$l_g\approx\lambda\!\!\!^{-}_u(\frac{\gamma J_A}{J})^{1/2}(\frac{\sigma_\perp}{\sigma_{cr}})^{2/3}\qquad(6)$$

at $\sigma_\perp>>\sqrt{\lambda\!\!\!^{-} l_g}$, where $\sqrt{\lambda\!\!\!^{-} l_g}$ is the *diffraction size* of the *self-amplified spontaneous emission* of FEL [15,16]. By comparison (5) and (6) one can find the critical transverse electron beam size σ_{cr}, at which $\sqrt{\lambda\!\!\!^{-} l_g}\approx\sigma_\perp$ [15]:

$$\sigma_{cr}=\frac{\lambda\!\!\!^{-}_u}{\gamma}(\frac{\gamma J_A}{J})^{1/4}.\qquad(7)$$

As exposed above, this parameter separates two different characteristic situations of SASE FEL – so-called cases of thin and wide beams.

BEAM TRANSPORT AND PHASING

An excitation caused by an ion at input will be transported together with electron beam developing exponentially independent of the ion; at the output from electron beam the ion acquire the momentum correlated with its input velocity. A firm correlation is possible unless the excitation reaches the nonlinear regime, i.e. the density modulation within the required scale of distances remains relatively small. It is, of course, necessary to provide the optimum output phase relations in the position, and velocity of an ion with respect to electron "avalanche" produced by the ion. Such a task is facilitated by the motion of ions and electrons in the fields given are absolutely different. In particular, after interaction at the "input" the beams can be separated and then can be made interacting again at the "output".

Compensation for Electron Delay

The gain process of CEC requires introduction of an alternating transverse magnetic field along the amplification section. This causes a decrease of electron translation velocity compared to the velocity of ion beam, hence, may lead to an ion run far away off the developed cloud of electron polarization initiated by the ion.. The picture is simple in case of an electrostatic instability used for the amplification, since in such a case the initiated polarization cloud does not propagate through the electron beam at equal absolute velocities of electron and ion beam ($\gamma_e=\gamma_h$), the related coherent delay of electrons can be compensated by divorcing two beams, and then introducing (or using) bend of the hadron beam, according to the condition

$$\int[\cos\alpha_e(s)-\cos\alpha_h(s)]ds=0,$$

or

$$<\alpha_e^2>\approx<\alpha_h^2>$$

where $\alpha_e(s)$, $\alpha_h e(s)$ are the electron and hadron orbit angle deviation from a straight line connecting the start (end of modulator) and finish (start of kicker) points of the electron bend. Other approach to compensation for a delay may consist of an increase of electron energy while avoiding bend of the hadron beam along straight section with continuous (helical) undulator (which covers all the CEC section), according to equation

$$\gamma_e^{-2}+\alpha_e^2=\gamma_h^{-2}$$

$$\gamma_e=\gamma_h/\sqrt{1-\gamma_h^2\alpha_e^2}$$

For example, $\gamma_e=\gamma_h\sqrt{2}$ at $\theta_u=1/(\gamma_h\sqrt{2})$. It should be noted that at a condition $\lambda\!\!\!^{-}_u\theta_u<\sigma_\perp$ the effective interaction force of ions with electrons in the modulator and kicker section will not decrease.

At use of SASE process for amplification, one has to take into account that the peak of electron polarization overtakes the translation motion of electrons in an undulator [17], that eases the compensation for electron delay.

An estimate of tolerances on static errors of the compensating field $b(s)$ leads to criterion as follow:

$$\delta s\cong\frac{\delta b}{b}<\alpha^2>\sqrt{Ll_{corr}}<\lambda\!\!\!^{-}\approx\frac{\lambda\!\!\!^{-}_u}{\gamma^2},$$

where L is effective length of section with bending dipoles (including the undulator section), and l_{corr} is the correlation length of the errors. In case of static errors $\delta s >> \lambda\!\!\!^-$, the residual mismatch can be compensated and controlled by a specific additional low dipole field.

Optimizing the Longitudinal Dispersion of Hadrons

After merging with electron beam in kicker section, an ion with momentum deviation *δp* from reference particle will have longitudinal displacement determined by the slip-factor α_s:

$$\Delta s \equiv |\alpha_s| L(\Delta p / p) \cong (\sigma_\perp / \gamma)$$

with $\alpha_s = \gamma^{-2} - <KD>$, where *K* and *D* are curvature and dispersion along reference orbit of ions.

An Optimal Focusing in Modulator and Kicker

At use of FEL mechanism for amplification of electron response to ions, which is adequate to high energy beams, the effective longitudinal length of polarization signal, $\sigma_\perp / \gamma$, produced by a single ion in the electron beam should not exceed the wave length of the FEL radiation:

$$\lambda\!\!\!^-_u \geq \gamma \sigma_\perp \quad (8)$$

From comparison between this condition and formula (7) for critical beam size associated with SASE's diffraction phenomenon we conclude that conditioning (8) can be realized only in case of "thin beam", when the diffraction size of FEL radiation exceeds the beam transverse size. Such optimization may require design of a low-beta focusing of ions in the modulator and kicker sections

Organizing the Transverse Cooling

Due to that possible gain mechanisms of CEC are naturally associated with the longitudinal interaction forces in electron beam, the excited polarization of electron space charge and related forces are in most longitudinal, as well. This circumstance would make transverse cooling ineffective compared to the longitudinal one, unless one implements a redistribution of cooling decrements. Similarly to re-distribution of the decrements of synchrotron radiation or electron cooling [12], it is possible to re-distribute decrements of CEC within the boundaries of the invariant sum of decrement. An effective method of re-distribution inherent to nature of CEC is proposed in work [10].

LIMITATIONS ON GAIN AND COOLING RATES OF CEC

Gain Limitation Due to Shot-noise of the Electron Beam

Apart from friction the particles will experience the scattering on the electron density fluctuations developing from the initial level at the input. Evaluations show that in the case of an unsuppressed Schottky-noise effect in the input (saturation regime of electron gun current), in this case the gain should not exceed the mass relation, in contrary the diffusion dominates over the friction [9]:

$$G < G_1 \equiv \frac{m_h}{m_e} \quad (9)$$

There are possibilities for suppressing the Schottky-noise [9].

A. It is well known that Schottky-noise is sharply decreased by collective interaction when operation the gun in the so-called "3/2 run". When accelerating electrons up to higher energies the beam (after leaving the "3/2" area) should be accelerated adiabatically in order to reach the further decreasing in the noise level.

B. Principle possibilities for decreasing the Schottky-noise effect exist also at operation of electron gun in saturation run. The idea consists in producing at the gain section input (where starts an exponential development of fluctuations) such phase relations in the noise in order to avoid amplification in the "over-heat" noise level (the heat oscillations cannot have the phase correlations). This appears feasible because of the boundary condition on the cathode for the Schottky-noise is the absence of the group fluctuations for electron velocities. This possibility is limited by the wave dispersion of plasma oscillation and inhomogeneity of the e-beam.

C. Finally, the longitudinal thermalization of plasma oscillation can be used for the suppression of Schottky-noise. In view of effective freezing of transverse electron motion by strong accompanying magnetic field, one can consider the electron beam as plasma with temperature equal to longitudinal temperature of the electron beam, which can be very low [13]. In such a situation the longitudinal plasma oscillations relax to thermodynamic level (corresponding to this temperature), which is a minimum among other shot-noise levels. Maintenance of that low shot-noise level along beam acceleration and transport before cooling section is an issue for study.

At suppression of shot-noise field by a factor Γ the admissible gain is increased by this factor:

$$G < G_\Gamma \equiv (m_h / m_e)\Gamma^2 \quad (10)$$

On the whole, the possibilities visible now for suppressing the Schottky-noise require their detail studies in the physical and technical aspects.

Gain Limits Due to Saturation of a Microwave Instability

Let us consider briefly the main limitations of an achievable increase for the response, due to non-linear saturation of instability. The most principal limitation is

$$G_{\max} < (n_e \sigma^3 / \gamma) \approx N_e (\sigma_\perp / \gamma \sigma_z).$$

due to a finite number of electrons participating effectively in the response.

The meaning of this limitation is self-evident: under this increase the deformation of an electron cloud of a size becomes (on the order of magnitude) unitary i.e. instability enters the nonlinear regime and an exponential evolution is ceased.

By taking into account the shot-noise of the electron beam, one has to reduce the achievable gain by the following one:

$$G < G_2 \equiv \Gamma(n_e \sigma_\perp^3 / \gamma)^{1/2}$$

By comparing this criterion with the previous one, we may conclude that maximum useful shot-noise suppression factor can be estimated as

$$\Gamma_{\max} \sim (n_e \sigma_\perp^3 / \gamma)^{1/2}$$

An additional limitation on gain is connected with "Schottky-noise" caused by the particles of the beam under cooling: since in the interaction region there are about $n_i \sigma_\perp^3 / \gamma$ ions, then

$$G < G_3 \equiv (n_e \sigma^3 / \gamma)/(n_i \sigma^3 / \gamma)^{1/2}$$

By reviewing the three estimated limitations, we can write a combined limitation on achievable gain:

$$G \le \min(G_2, G_3, \Gamma^2 \frac{m_p}{m_e})$$

The Shield Effect in CEC

There is a limitation on cooling time in method of *stochastic cooling* due to the shielding interaction of ions via the amplifier [17]:

$$\tau_c \ge \frac{(J_p / e)}{(\Delta\omega)^2} \frac{f_0}{\Delta f_0}$$

Where J_p is the current of a (coasted) beam under cooling, Δf_0 is the spread of particle revolution frequency f_0,

$$\Delta\omega = 2\pi\Delta f \le (c / l_\perp) \qquad (11)$$

is the penetration frequency bandwidth of an amplifier, and $l_\perp$ is an effective aperture of the pickup-kicker electrodes. When considering a correspondent limitation on cooling time of CEC, we have to substitute the frequency bandwidth in (11) as

$$\Delta\omega = \frac{\gamma\beta c}{\sigma_\perp}.$$

One can see that the shield limitation of CEC to be substantially weaker than in ordinary stochastic cooling. The difference is especially big in ultra relativistic region. This limitation seems to be insignificant even when cooling very short bunches in colliders. When cooling at low energies, it is important that this limitation does not increase but decrease with the cooling process. In particular, when stacking, the cooling of a new portion of particles is not essentially decelerated by the presence of an already stored intense beam that is different from the case of stochastic cooling. In this aspect the method suggested here maintains in practice the properties of an ordinary electron cooling method.

To finish our general observation of the method, note the shielding effect is related to the question of collective stability beam under cooling which also has to be studied as a possible limiting factor.

CONCLUSIONS AND OUTLOOK

The method considered above combines principles of electron and stochastic cooling and microwave amplification using an electron beam. Such unification promises to frequently increase the cooling rate compared to both the electron cooling and conventional stochastic cooling. It might find important applications to projects based on cooling and stacking of high-temperature, intense heavy particle beams in a wide energy range.

Some tentative schematics and estimations of cooling rates of CEC that could be used for luminosity increase in colliders with hadron beams are presented in work [10]. The preliminary results are encouraging. Certainly, for the whole understanding of new possibilities thorough theoretical study is required of all principle properties and other factors of the method.

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